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First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1131.1 : MATHEMATICS I – CALCULUS AND SEQUENCES AND SERIES

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 80

I. Answer the first **ten** questions are compulsory. They carry **1** mark each.

1. Find $\lim_{x \to 1}\left(x^7 - 2x^5 + 1\right)^{35}$.

2. What is the value of $\lim_{x \to -\infty} \tan^{-1} x$?

3. Evaluate $\int\left(x + x^2\right)dx$.

4. What is the integral of $\tan x$?

5. Find $\int_0^{\pi/2} \frac{\sin x}{5}\,dx$.

6. Find the area under the curve $y = \sin x$ over the interval $[0, \pi/4]$.

7. Find $\frac{\partial f}{\partial y}$ for the function $f(x, y) = 2x^3y^2 + 2y + 4x$.

P.T.O.

8. Define critical point.

9. Find the general term of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

10. Find the Maclaurin polynomial P_2 for e^x.

(10 × 1 = 10 Marks)

II. Answer any **eight** questions. They questions carry **2** marks each.

11. Find $\lim_{x \to -4} \frac{2x + 8}{x^2 + x - 12}$.

12. For what values of x is there a discontinuity in the graph of $y = \frac{2x + 3}{(x - 5)(x - 6)}$?

13. Find $\frac{dy}{dx}$ if $y = \sec^{-1}(e^x)$.

14. Evaluate $\int \frac{t^2 - 2t^4}{4} dt$.

15. Evaluate $\int \frac{dx}{1 + 3x^2}$.

16. Evaluate $\int \cos^2 x \, dx$.

17. Describe the level surfaces of $f(x, y, z) = z^2 - x^2 - y^2$.

18. If $f(x, y) = x^2y^3 + x^4y$, find $\frac{\partial^2 f}{\partial y^2}$.

19. Consider the sphere $x^2 + y^2 + z^2 = 1$. Find $\frac{\partial z}{dx}$ and $\frac{\partial z}{\partial y}$ at the point $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.

20. Determine whether the sequence $\left\{(-1)^{n+1} \frac{n}{2n + 1}\right\}_{n=1}^{+\infty}$ converges or diverges.

21. Determine whether the series $\sum_{k=1}^{\infty} 3^{2k}\, 5^{1-k}$ converges or diverges.

22. Show that the series $\sum_{k=1}^{\infty} \frac{k}{k+1}$ diverges.

(8 × 2 = 16 Marks)

III. Answer any **six** questions. These questions carry **4** marks each.

23. Find $\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1}$.

24. Find $\lim_{x \to -\infty} \frac{4x^2 - x}{2x^3 - 5}$.

25. Evaluate $\int x e^x \, dx$.

26. Evaluate $\int_{1}^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$.

27. Let $f(x,y) = y^2 e^x + y$. Find f_{xyy}.

28. Given that $z = e^{xy}, x = 2u + v, y = u/v$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain rule.

29. Locate all relative extrema and saddle points of $f(x,y) = 4xy - x^4 - y^4$.

30. Show that the integral test applies and use the integral to determine whether the series $\sum_{k=1}^{\infty} \frac{1}{k^2}$ converge or diverge.

31. Use the comparison test to determine whether the series $\sum_{k=1}^{\infty} \frac{1}{2k^2 + k}$ converge or diverge.

(6 × 4 = 24 Marks)

IV. Answer any **two** questions. These questions carry **15** marks each.

32. (a) Find $\frac{dy}{dx}$ if $y = 3x^8 - 2x^5 + 6x + 1$.

(b) At what points, does the graph of $y = x^3 - 3x + 4$ have a horizontal tangent line?

(c) Find the area of the triangle formed from the coordinate axes and the tangent line to the curve $y = 5x^{-1} - \frac{1}{5}x$ at the point $(5, 0)$.

33. (a) Evaluate $\int \sin^4 x \cos^4 x \, dx$.

(b) Evaluate $\int \tan^2 x \sec^4 x \, dx$.

34. Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of 32 ft^3 and requiring the least amount of material for its construction.

35. (a) Find the n^{th} Maclaurin polynomial for $\frac{1}{1-x}$ and express it in sigma notation.

(b) Find the n^{th} Taylor polynomial for $\frac{1}{x}$ about $x = 1$ and express it in sigma notation.

(2 × 15 = 30 Marks)